

Illiquidity frictions and asset pricing anomalies

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Abstract

This paper investigates the relation between illiquidity level and illiquidity risk and the size, value and momentum anomalies for US stocks 1931-2012. In contrast to statistical factors both illiquidity level and illiquidity risk have a solid theoretical foundation in the liquidity adjusted capital asset pricing model (LCAPM). A horse race between the LCAPM and the CAPM shows that the LCAPM outperforms the CAPM in terms of ability to explain risk premiums of size and value sorted test portfolios. The outrun comes mostly from the illiquidity level premium while the illiquidity risk premium is overall less important. We find a very strong correlation between Fama-French size betas and illiquidity level betas (about 0.96) and a fairly strong correlation between Fama-French value betas and illiquidity risk betas (about 0.56) while Carhart's momentum beta has high negative correlation with betas both for illiquidity level and risk (-0.76 and -0.94 respectively). The premiums related to size can to large extent be explained as a compensation for illiquidity level. The premiums related to momentum are essentially explained by the high illiquidity (both level and risk) of the portfolios with little momentum. The premiums related to value can to some extent be interpreted as compensation for illiquidity risk and to some extent as compensation for illiquidity level but cannot fully explain the value anomaly.

Key words: illiquidity frictions, asset pricing anomalies, CAPM, LCAPM
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1. Introduction

The liquidity adjusted capital asset pricing model (LCAPM) developed by Acharya and Pedersen (2005), suggests that illiquidity level and illiquidity risk are fundamental factors in asset returns, in addition to the market factor in the Sharpe (1964) and Lintner (1965) capital asset pricing model (CAPM). Different versions the LCAPM are estimated in previous literature, but there is no direct comparison between the two models and therefore no judgement of whether augmenting the frictionless CAPM economy with frictions in the form of stochastic trading costs actually helps from an empirical perspective. The main purpose of our paper is to investigate this question.

We construct traded factor portfolios representing illiquidity level and illiquidity risk employing an independent double sort procedure on level and risk similar to Fama and French (1993). We examine in detail the ability of the LCAPM to explain the averages of US stock returns from 1931-2012. Only Pástor and Stambaugh (2003) and Liu (2006) have used traded factors. Pástor and Stambaugh (2003) construct a factor that is close to our illiquidity risk factors for the second channel i.e. the asset return comovement with market illiquidity, which is then used for a mean-variance analysis. Liu (2006) constructs a factor for the illiquidity level, which is based on his liquidity

measure the standardized turnover-adjusted number of zero daily trading volumes over the prior 12 months. Lou and Sadka (2011) use a double-sorting that is akin to ours, but they do not construct factors that are used in an asset pricing exercise. Thus, none have used traded factors both for the illiquidity level and the illiquidity risk.

Our factors provide a means to investigate the closely connected question of whether illiquidity level or illiquidity risk is more important from an asset pricing perspective. Besides Acharya and Pedersen (2005) only Sadka et al. have investigated both illiquidity level and illiquidity risk (see Sadka (2006), Korajczyk & Sadka (2008), Lou & Sadka (2011)). However, Sadka et al. use completely different illiquidity measures. While most of the related liquidity literature focuses on commonality risk, we construct traded factors for all three channels through which illiquidity risk can affect asset prices in the LCAPM; asset illiquidity comovement with market illiquidity (commonality risk), asset return comovement with market illiquidity and finally asset illiquidity comovement with market return. This allows us to empirically investigate potential differential influences on asset prices through the different channels.

Finally we investigate the relation between our liquidity factors and the

well-known factors representing size (SMB), and value (HML) and momentum (MOM) as constructed by Fama and French.

The results show that the LCAPM outperforms the CAPM in terms of ability to explain risk premiums of size and value sorted test portfolios. Furthermore, the illiquidity level premium is mostly more important than the illiquidity risk premium. There is a very strong correlation between Fama-French size betas and illiquidity level betas; a fairly strong correlation between Fama-French value betas and illiquidity risk betas; Carhart's momentum beta has high negative correlation with betas both for illiquidity level and risk. The premiums related to size can to large extent be explained as a compensation for illiquidity level. The premiums related to momentum are essentially explained by the high illiquidity (both level and risk) of the portfolios with little momentum. The premiums related to value can to some extent be interpreted as compensation for illiquidity risk and to some extent as compensation for illiquidity level but cannot fully explain the value anomaly. These results are robust in subsamples and to the specific channel of illiquidity risk considered.

The structure of the paper is the following: section 2 discusses the implementation of the LCAPM; section 3 shows how the factors are constructed

and their properties; section 4 contains the result; the paper ends with a short conclusion.

2. Implementation of the LCAPM

The LCAPM by Acharya and Pedersen (2005) is similar to the CAPM in the sense that risk averse agents maximize expected utility in a one-period framework. The key LCAPM assumption is that there are frictions in the economy in the form of stochastic illiquidity costs. The LCAPM asset pricing relation is derived by adjusting returns for illiquidity costs and assuming that the CAPM holds for adjusted returns (returns net of illiquidity costs). This brings about an asset pricing relation with a premium for illiquidity level and three types illiquidity risk premiums besides the traditional market risk premium. The different types of illiquidity risk are:

β_1 : covariance between asset illiquidity and market illiquidity,

β_2 : covariance between asset return and market illiquidity,

β_3 : covariance between asset illiquidity and market return.

The three betas represent the different channels through which illiquidity risk can influence asset prices in the LCAPM, with β_1 reflecting commonality risk (Chordia, Roll and Subrahmanyam, 2000). Commonality risk is the risk of

holding a security that becomes illiquid when the market in general becomes illiquid. Illiquidity risk related to β_2 is the risk of holding an asset with a low return in times of market illiquidity (Pástor and Stambaugh, 2003) and β_3 represents illiquidity risk that leads to a premium for holding an asset that is illiquid in low return states of the market. For most stocks we therefore expect $\beta_0 > 0$, $\beta_1 > 0$, $\beta_2 < 0$ and $\beta_3 < 0$, where β_0 is the traditional market beta.

We implement the LCAPM in terms of traded factors (for asset i and time t):

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{LLIQ}^i LLIQ_t + \beta_{RLIQ}^i RLIQ_t + \varepsilon_t^i, \quad (1)$$

where MKT represents a market risk factor, $LLIQ$ is a traded factor representing illiquidity level and $RLIQ$ is a traded factor representing illiquidity risk. We construct traded factors for all three sources of illiquidity risk. Our implementation of the CAPM naturally restricts the betas on $LLIQ$ and $RLIQ$ in Eq. (1) to zero. The next section describes the construction of the traded factors.

3. Factor construction and factor properties

We first independently double sort the universe of eligible stocks on illiquidity level and illiquidity risk (Subsection 3.1). These underlying portfolios form the basis for the construction of the traded factor portfolios LLIQ and RLIQ (Subsection 3.2). In the LCAPM, the illiquidity level component of expected return is the expected relative illiquidity cost over the holding period. We therefore use the effective spread as our illiquidity measure; the effective spread is a direct estimate of the expected relative illiquidity cost. We employ the Fong, Holden and Trzcinka (2011) low frequency proxy based on daily return observations. They show that their measure (FHT) outperforms all other low frequency proxies for most international stock markets, including the US stock market, both in terms of ability to reproduce the level and the variability of the "true" effective spread estimated from high frequency trades and quotes data.

3.1. Underlying portfolios

We construct four underlying portfolios by independently sorting eligible stocks on illiquidity level (low/high) and on illiquidity risk (low/high). This sorting procedure is repeated for each of the three different types of illiquid-

ity risk in the LCAPM; asset illiquidity comovement with market illiquidity (commonality risk, β_1), asset return comovement with market illiquidity (β_2) and asset illiquidity comovement with market return (β_3). Portfolio formation is yearly and we calculate monthly returns for the underlying portfolios for the year following formation. We use common NYSE and Amex stocks in the CRSP database with a share code of 10 and 11 and a share price of at least one dollar and at most 1000 dollar. To calculate the FHT effective spread proxy for an individual stock we require at least 100 daily return observations any given year. The illiquidity level of a stock in a given year is then taken as a forecast of the illiquidity level of the stock in the next year. Illiquidity risk is measured as illiquidity beta, where the different illiquidity betas are OLS-estimates from five-year rolling window regressions. We require at least 30 monthly observations in any given five-year window. The dependent variables in the regressions are innovations in individual stock illiquidity level (for estimating β_1 and β_3) or individual stock return (β_2) and the explanatory variables are innovations in market illiquidity level (β_1, β_2) or market return (β_3). These regressions essentially follow the calculation of unconditional betas in Acharya and Pedersen (2005). The illiquidity risk of a stock estimated over a given five-year window is taken as a forecast

of illiquidity risk of the stock in the next year. Each year all stocks with the economically expected sign of the beta forecast are used in the portfolio construction. Hasbrouck (2009) argues that imposing arbitrary weights after sorting stocks into portfolios is suspicious of distorting the original sorting. We therefore focus our analysis on equal weighted portfolios. Table 1 shows summary statistics for the underlying portfolios for each of the three different measures of illiquidity risk over the full sample period and the subsample periods 1931-1964 and 1965-2012.

Table 1

The table shows monthly returns and standard deviations for three sets of underlying portfolios. In the first set of portfolios illiquidity risk is defined as asset illiquidity comovement with market illiquidity (commonality risk, β_1). In the second set of portfolios illiquidity risk is defined as asset return comovement with market illiquidity (β_2). In the third set of portfolios illiquidity risk is defined as asset illiquidity comovement with market return (β_3). *LL* (*HL*) denotes low (high) illiquidity level and *LR* (*HR*) denotes low (high) illiquidity risk.

		β_1		β_2		β_3		
		<i>LR</i>	<i>HR</i>	<i>LR</i>	<i>HR</i>	<i>LR</i>	<i>HR</i>	
1931-2012	Return	<i>LR</i>	1.119	1.500	1.102	1.470	1.141	1.503
		<i>HR</i>	1.190	1.636	1.221	1.600	1.265	1.616
	Std.	<i>LR</i>	6.035	7.934	5.780	7.660	6.153	8.343
		<i>HR</i>	7.442	9.397	7.722	9.436	7.533	9.225
1931-1964	Return	<i>LR</i>	1.206	1.742	1.114	1.758	1.179	1.763
		<i>HR</i>	1.275	1.951	1.350	1.908	1.423	1.944
	Std.	<i>LR</i>	7.488	9.754	6.971	9.490	7.653	10.494
		<i>HR</i>	9.334	11.710	9.413	11.602	9.546	11.425
1965-2012	Return	<i>LR</i>	1.057	1.328	1.094	1.265	1.114	1.319
		<i>HR</i>	1.130	1.413	1.130	1.382	1.153	1.383
	Std.	<i>LR</i>	4.751	6.341	4.766	6.038	4.823	6.400
		<i>HR</i>	5.745	7.335	6.262	7.539	5.699	7.277

The underlying portfolios show very similar patterns for return and standard deviation for all for all three liquidity betas. Ranking portfolios by either return or standard deviation return always give the same portfolio order. The high level portfolios always have higher returns and standard deviations than the corresponding low level portfolios. The high risk portfolios always have higher returns and standard deviations than the corresponding low risk portfolios. The lowest return and standard deviation is always for the low level/low risk portfolio. The highest return and standard deviation is always for the high level/high risk portfolio. This pattern is very consistent for all three samples (subsamples). It is also clear that both return and standard deviation for all portfolios are consistently lower for the second part of the sample.

3.2. Illiquidity level factor and illiquidity risk factor

The traded factors are high-minus-low portfolios for illiquidity level and illiquidity risk calculated using the four underlying portfolios. Denoting returns on the underlying portfolios by low level/low risk (LL/LR), low level/high risk (LL/HR), high level/low risk (HL/LR) and high level/high risk (HL/HR), returns on the traded illiquidity level factor ($LLIQ$) and the

traded illiquidity risk factor ($RLIQ$) are calculated as:

$$LLIQ = \frac{1}{2} (HL/LR + HL/HR) - \frac{1}{2} (LL/LR + LL/HR), \quad (2)$$

$$RLIQ = \frac{1}{2} (LL/HR + HL/HR) - \frac{1}{2} (LL/LR + HL/LR). \quad (3)$$

The returns on these factors are denoted by $LLIQ_1$, $RLIQ_1$, $LLIQ_2$, $RLIQ_2$, $LLIQ_3$ and $RLIQ_3$, where the index indicates the type of illiquidity risk in the LCAPM. Table 2 shows summary statistics for the traded factors for each of the three different sources of illiquidity risk over the full sample period and the subsample periods 1931-1964 and 1965-2012.

Table 2

The table shows monthly returns, standard deviations and correlations for three sets of traded factors. In the first set of factors illiquidity risk is defined as asset illiquidity comovement with market illiquidity (commonality risk, β_1). In the second set of factors illiquidity risk is defined as asset return comovement with market illiquidity (β_2). In the third set of factors illiquidity risk is defined as asset illiquidity comovement with market return (β_3). Returns on the corresponding illiquidity level and illiquidity risk factors are denoted by $LLIQ_1$, $RLIQ_1$, $LLIQ_2$, $RLIQ_2$, $LLIQ_3$ and $RLIQ_3$.

		$LLIQ_1$	$RLIQ_1$	$LLIQ_2$	$RLIQ_2$	$LLIQ_3$	$RLIQ_3$
1931-2012	Return	0.413	0.104	0.373	0.125	0.356	0.119
	Std.	3.652	1.949	3.667	2.444	3.778	1.705
1931-1964	Return	0.605	0.139	0.601	0.193	0.552	0.213
	Std.	4.241	2.441	4.384	2.953	4.448	2.112
1965-2012	Return	0.277	0.079	0.211	0.076	0.217	0.052
	Std.	3.168	1.510	3.055	2.010	3.219	1.343
1931-2012	Corr.						
		$LLIQ_1$	$RLIQ_1$	$LLIQ_2$	$RLIQ_2$	$LLIQ_3$	$RLIQ_3$
	$LLIQ_1$	1					
	$RLIQ_1$	0.663	1				
	$LLIQ_2$	0.970	0.702	1			
	$RLIQ_2$	0.547	0.730	0.485	1		
	$LLIQ_3$	0.965	0.711	0.965	0.531	1	
	$RLIQ_3$	0.577	0.746	0.568	0.699	0.509	1

Returns on the factor portfolios representing illiquidity level ($LLIQ_1$, $LLIQ_2$ and $LLIQ_3$) are as expected almost perfectly correlated. The factor premium is also similar; on average 0.38% per month for the full sample. For subsamples, the illiquidity level factor premium decreases over time; from on average 0.59% per month for the first subperiod to 0.24% per month for the second subperiod. Returns on the factor portfolio representing β_1 -risk ($RLIQ_1$) is 0.10% per year for the full sample. Corresponding factor premiums for portfolios representing β_2 -risk ($RLIQ_2$) and β_3 -risk ($RLIQ_3$) are slightly larger; around 0.12% per month. Again there is a clear decrease in premium between the first and the second subsample. For the first sample period illiquidity risk factor premiums are on average 0.18% and for the second sample period 0.07% per month. We note that illiquidity level factors have higher premiums than illiquidity risk factors. As a consequence it is more likely that the illiquidity level factor is useful for explaining the variation in the cross-section of average stock return (Fama and French, 1993). For explaining the common variation in returns the story is different; in the extreme case a factor can have a zero premium but still have a significant loading (Cochrane, 2001). Correlations between level factors and correspond-

ing risk factors are positive and is the highest between $LLIQ_1$ and $RLIQ_1$ (0.66) and the lowest between $LLIQ_2$ and $RLIQ_2$ (0.48). This positive correlation between level and risk is not surprising; see, e.g., Hagströmer, Hansson and Nilsson (2013), who find a correlation between illiquidity level premium and illiquidity risk premium around 0.35.

4. Results

The bootstrap procedure described below is our primary tool for comparing the performance of the LCAPM and the CAPM. The test portfolios employed are 25 portfolios independently double sorted on market capitalization and book-to-market ratio and 10 portfolios sorted on momentum.¹ We further group test portfolios in five size groups with increasing market capitalization and in five value groups with increasing book-to-market (with five portfolios in each group).² Finally, we perform the bootstrap tests for all

¹We are grateful to Kenneth French for making these portfolio returns available at: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. We use both equal weighted and value weighted versions of the test portfolios.

²Retaining the original ordering we number the test portfolios P1-P25. We denote the five size groups as S1: P1-P5, S2: P6-P10, S3: P11-P15, S4: P15-P20 and S5: P21-P25. The five value groups are denoted V1: P1, P6, P11, P16, P21, V2: P2, P7, P12,

25 portfolios (in a single group). These test portfolios are often employed in previous research. In addition, the underlying sorting variables are different from illiquidity, which creates a more level playing field for comparing the LCAPM and the CAPM. The bootstrap procedure is as follows. 1. Generate 10000 bootstrap samples. 2. Estimate the regression in Eq. (1) with OLS for each bootstrap sample and each test portfolio and record parameter estimates for both the LCAPM and the CAPM. 3. Count the number of regressions for each portfolio for which the LCAPM performs better than the CAPM, i.e. the number of times that the intercept is lower for the same bootstrap sample (pairwise comparison). 4. Perform a one-sided binomial test at the 95% confidence level to conclude whether the LCAPM performs significantly better or significantly worse than the CAPM (or if there is no statistically significant difference).³ The tests for groups of portfolios are performed by calculating averages within groups and counting the number of times the within-group alpha is lower for the LCAPM than for the CAPM. We also

P17, P22, V3: P3, P8, P13, P18, P23, V4: P4, P9, P14, P19, P24 and V5: P5, P10, P15, P20, P25. Therefore, S1 contains small capitalization stocks and S5 contains large capitalization stocks. Similarly, V1 contains low book-to-market stocks and S5 contains high book-to-market stocks.

³The critical values at the 5% level for the one-sided binomial test are 4918 and 5082.

report bootstrapped point estimates of regression coefficients and construct 95% confidence intervals for these estimates. We first investigate whether LCAPM improves upon the CAPM (Subsection 4.1); secondly, the contributions to risk premiums from the illiquidity level factor and the illiquidity risk factor (Subsection 4.2); thirdly, the anomalies (size, value, momentum) in terms of illiquidity premiums (Section 4.3)

4.1. Interpreting size and value anomalies in terms of illiquidity premiums

Both the LCAPM and the CAPM implies that the intercept (alpha) in Eq. 1 is zero (see, e.g., Fama and French, 2004). To compare the models in this respect we analyze the bootstrapped intercepts. Table 4 reports the results of a pairwise comparison between alphas from the LCAPM and the CAPM. If the LCAPM produces a lower absolute alpha in more than 5082 regressions (out of 10000), the LCAPM significantly surpass the CAPM. This happens for roughly 18 portfolios over the full sample period with only small variations between the different illiquidity risk channels. There are just minor differences between equal and value weighted test portfolios. The results for value weighted portfolios over the first subperiod 1931-1964 differ a bit in that the superior performance of the LCAPM is slightly less clear. For the second subperiod 1965-2012, the outperformance of the LCAPM is even

stronger than for the full sample period, both for equal weighted and value weighted test portfolios. Figure 1 shows the number of regressions for which the LCAPM gives a lower absolute alpha than the CAPM.⁴ The LCAPM substantially outperforms the CAPM for most of the smaller stock portfolios, the exceptions are portfolios with small growth stocks; in particular P1, P2, P6 and P11. The CAPM is also doing better than the LCAPM for some of the big stock portfolios; in particular P22-P24.

Figure 1 approximately here.

Grouping the portfolios according to size and book-to-market gives a clearer picture of when the LCAPM outperforms the CAPM and vice versa. The results for three different groupings - all portfolios, five size and five value groups respectively - and two subperiods - 1931-1964 and 1965-2012 - are presented in Figure 2. The LCAPM performs overall better than the CAPM for all size groups, from small capitalization stocks in group S1 to large capitalization stocks in group S5. The outperformance is again even stronger for the second subperiod 1965-2012. For the value groups we find that the

⁴We only present a figure for the commonality channel. As is evident from Table 4, the corresponding figures for the other two illiquidity channels look very similar.

LCAPM outperforms the CAPM for most value groups, in particular for the high book-to-market portfolios (value stocks) in groups V3-V5. The notable exception is that the CAPM is clearly superior over both the full sample period and the subperiods for the low book-to-market portfolios (growth stocks) in group V1.

Figure 2 approximately here.

The bootstrap point estimates and confidence interval of alpha for the LCAPM and the CAPM are presented in Figure 3A-B. Even though the LCAPM outperforms the CAPM according to the binomial test, it is clear that neither the LCAPM nor the CAPM is a complete asset pricing model in the sense that about half of the intercepts are significant for both models. There are no systematic differences between equal weighted test portfolios and value weighted test portfolios. The pattern for alpha is that going from growth to value portfolios (from P1 to P5, from P6 to P10, etc.), the magnitude of the intercept is mostly increasing for both LCAPM and CAPM, indicating that neither model is able to entirely capture the value effect. Going from small stocks to large stocks, the intercept is increasing with size, indicating that neither of the models is able to fully capture the size effect.

Figure 3A-B approximately here.

Table 4

The table shows the number of equal weighted and value weighted test portfolios for which the LCAPM performs significantly better than the CAPM in terms of alpha (and vice versa). The three different channels of illiquidity risk are denoted β_1 , β_2 and β_3 . The table is based on the binomial test performed on 10000 bootstrap samples; the test is described in more detail in footnote 3.

		Equal weighted		Value weighted	
		LCAPM	CAPM	LCAPM	CAPM
1931-2012	β_1	17	8	18	7
	β_2	17	8	17	8
	β_3	19	6	17	8
1931-1964	β_1	19	6	16	8
	β_2	16	7	11	13
	β_3	15	9	14	11
1965-2012	β_1	20	5	19	6
	β_2	16	8	18	7
	β_3	17	7	17	6

4.2. Contributions from illiquidity factors to portfolio risk premiums

In this subsection we analyze the contributions to total portfolio risk premium from the illiquidity level factor and the illiquidity risk factor (and the market factor). The contribution from each of the factors is calculated according to Eq.1, i.e., as portfolio beta times the factor premium. Figures 4A-B, 5A-B and 6A-B show estimated (bootstrapped) betas for the market factor, LLIQ (illiquidity level factor) and RLIQ (illiquidity risk factor) for each of the test portfolios. We note that market betas are always closer to one for the LCAPM compared to the CAPM, suggesting that the market factor in the CAPM to some extent is able to act as a substitute for the illiquidity factors. The illiquidity level betas decrease with size and increase with value, indicating that the illiquidity level factor is more important for small stocks and for value stocks. The liquidity risk betas show the same pattern as the illiquidity level betas both with respect to size portfolios and value portfolios. Magnitudes of the two types of illiquidity betas are quite similar, but the RLIQ betas are overall estimated with less precision. The equal weighted and the value weighted test portfolios exhibit the same patterns, but the magnitudes of betas are in general slightly smaller for value weighted test

portfolios.⁵

Figures 4A-B, 5A-B and 6A-B approximately here.

To shed light on the issue of the relative economic importance of LLIQ and RLIQ we analyze their contribution to average portfolio excess return (the risk premium). Figure 7A-B shows the contributions in monthly percentage terms from the illiquidity level factor and the illiquidity risk factor. Apparently, the contributions from the two factors follow the same pattern as the betas themselves, i.e., contributions are decreasing with size and increasing with value. Comparing LLIQ with RLIQ, the contribution from the illiquidity level factor is almost always larger than the contribution from the illiquidity risk factor. This is a result of the lower factor premium for RLIQ than for LLIQ. For the first five portfolios (group S1) the total illiquidity premium, i.e., level premium plus risk premium is quite large, on average around 0.55% per month, of which the illiquidity level premium is about 0.44% and the illiquidity risk premium is about 0.12%. For the large stock portfolios (group S5) the contributions are markedly smaller from both illiquidity fac-

⁵Estimated betas for value weighted test portfolios are available upon request from the authors.

tors. The illiquidity factors contribute more to the risk-premiums of high book-to-market portfolios than of low book-to-market portfolios. Group V5 (value stocks) carry an illiquidity premium of on average about 0.32% per month and group V1 (growth stocks) carry a premium of on average about 0.20%. To gauge the overall economic importance of our risk factors we compare in Figure 8A-B their absolute contribution to average return relative to the contribution of the market portfolio, i.e., we divide the absolute contribution from the two illiquidity factors with the (absolute) contribution from the market factor. Again, the pattern is a decreasing importance of the illiquidity factors with size and an increasing importance with value. The liquidity factors are quite important compared to the market factor for the first fifteen portfolios (groups S1-S3). For group S1 the contribution to the risk-premium from the illiquidity factors are about 80% of the contribution from the market factor, for group S2 it is about 40% and for group S3 about 20%.

Figures 7A-B and 8A-B approximately here.

4.3. Interpreting anomalies in terms of illiquidity premiums

Once we have our factors it is interesting to see how they are related to the well known factors constructed by Fama and French, Size (SMB) and

Value (HML), and Carhart's momentum factor (MOM) (Carhart 1997).⁶

The contribution to the risk premium of a test portfolio is the product of the beta values (sensitivities) times the mean return of the factors. It is obvious that these factors are better to explain the risk premium for our 25 test portfolios or the 10 momentum sorted portfolios, since they are tailor-made for this purpose. As can be seen from the Table 5 below the mean return of the liquidity level factor (LLIQ3) is on parity with SMB and HML while the liquidity risk factor (RLIQ3) is lower.

⁶We take the factors from French's homepage.

Table 5

The table shows average return, standard deviation and the t -value for the average return being different from zero for the traded factors from 193101-201212.

	Mkt-Rf	SMB	HML	MOM	LLIQ3	RLIQ3
Average	0.645	0.290	0.434	0.611	0.356	0.119
Std	5.382	3.296	3.596	4.824	3.778	1.705
t	3.758	2.756	3.784	3.971	2.957	2.182

The beta values for SMB and HML from the FF3 model (25 portfolios) and MOM from FF4 (10 portfolios) are closely related to the beta values from LCAPM (see Figure 9A-C). It is evident that SMB and LLIQ3 are highly correlated (0.96) and, furthermore, since the average value of LLIQ is close to SMB, it follows that the size anomaly can to large extent be explained as a compensation for illiquidity level. MOM is highly negatively correlated both to LLIQ3 (-0.76) and RLIQ3 (-0.94), which also implies that the momentum anomaly is mainly related to the high illiquidity (both level and risk) of the portfolios with little momentum (approximately P1 to P3 in figure). HML is correlated both to RLIQ3 (0.56) and LLIQ3 (0,40), and the value anomaly is therefore to some extent a compensation for illiquidity.

Figures 9A-C approximately here.

5. Summary and concluding remarks

The relative comparison between the LCAPM and the CAPM implemented in this paper shows that the LCAPM outperforms the CAPM in its ability to explain the cross-section of average returns. From an absolute perspective it is however clear the neither the LCAPM nor the CAPM is a complete asset pricing model, since none of the models fully capture neither

the size effect nor the value effect.

This paper has investigated the relation between illiquidity level and illiquidity risk and the size, value and momentum anomalies for US stocks 1931-2012. The theoretical starting point is the liquidity adjusted capital asset pricing model (LCAPM), since this model gives a solid theoretical foundation both to illiquidity level and illiquidity risk, which is in contrast to statistical factors such as the well-known factors: size (SMB), value (HML) and momentum (MOM). However, we first investigate whether LCAPM has anything to offer in comparison with CAPM. A horse race between the LCAPM and the CAPM shows that the LCAPM outperforms the CAPM in terms of ability to explain risk premiums of size and value sorted test portfolios. The outrun comes mostly from the illiquidity level premium while the illiquidity risk premium is overall less important. To investigate the relation between our liquidity factors and size, value and momentum we look at the relation between their beta values. We find a very strong correlation between Fama-French size betas (SMB) and illiquidity level betas (LLIQ) (about 0.96) and a fairly strong correlation between Fama-French value betas (HML) and illiquidity risk betas (RLIQ) (about 0.56) while Carhart's momentum betas (MOM) have high negative correlation with betas both for illiquidity level

and risk (-0.76 and -0.94 respectively). The premiums related to size can to large extent be explained as a compensation for illiquidity level. The premiums related to momentum are essentially explained by the high illiquidity (both level and risk) of the portfolios with little momentum. The premiums related to value can to some extent be interpreted as compensation for illiquidity risk and to some extent as compensation for illiquidity level but cannot fully explain the value anomaly.

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